4734 Probability & Statistics 3

1(i)	$\int_{-a}^{0}\frac{2}{5}\,dx+\int_{0}^{\infty}\frac{2}{5}e^{-2x}dx=1$ $2a/5 + 1/5 = 1$ $a = 2$	M1 A1 A1 **3**	Sum of probabilities =1
(ii)	$\int_{-2}^{0}\frac{2}{5}x\,dx+\int_{0}^{\infty}\frac{2}{5}xe^{-2x}dx$ $\int_{-a}^{0}\frac{2}{5}x\,dx=-\frac{a^2}{5}$ $\int_{0}^{\infty}\frac{2}{5}xe^{-2x}dx=\left[-\frac{1}{5}xe^{-2x}\right]+\left[-\frac{1}{10}e^{-2x}\right]$ $= - 0.7$	M1 A1 √ M1 A1 A1 **5** **[8]**	$\Sigma \int x\, f(x)\, dx$ $\sqrt{a}$ By parts with 1 part correct Both parts correct CAO
2(i)	4 cartons: Total, $Y \sim N(2016, 36)$ $P(Y \leq 2000) = \Phi(-16/\sqrt{36})$ $= 0.00383$	B1B1 M1 A1 **4**	Mean and variance
(ii)	$E(V) = 0$ $Var(V) = 36 + 16\times 9$ $= 180$	B1 M1 A1 **3**	CWO
(iii)	0.5	B1 **1** **[8]**	
3(i)	Normal distribution Mean $\mu_1 - \mu_2$; variance $2.47/n_1 + 4.23/n_2$	B1 B1B1 **3**	
(ii)	$H_0: \mu_1 = \mu_2$, $H_1: \mu_1 \neq \mu_2$ $(9.65 - 7.23)/\sqrt{(2.47/5+4.23/10)}$ $=2.527$ > 2.326 Reject H_0 There is sufficient evidence at the 2% significance level that the means differ	B1 M1 B1 A1 M1 A1 **6**	Or find critical region Numerator Compare with critical value **SR1:If no specific comparison but CV and conclusion correct B1. Same in Q5,6,7** **SR2: From CI: $2.42 \pm z\sigma$ M1, σ correct** **z = 2.326 B1, (0.193, 4.647) A1** **0 in not in CI ; reject H_0 etc M1A1 Total 6** **Conclusions not over-assertive in Q3, 5, 6**
(iii)	Any relevant comment.	B1 **1** **[10]**	e.g sample sizes too small for CLT to apply

4(i)	$G(y)=P(Y \le y)=P(1/(1+V) \le y)$ $=P(V \ge 1/y - 1)$ $= 1 - F(1/y - 1)$ $= \begin{cases} 0 & y \le 0, \\ 8y^3 & 0 < y \le 1/2, \\ 1 & y > 1/2. \end{cases}$ $g(y) = \begin{cases} 24y^2 & 0 < y \le 1/2, \\ 0 & \text{otherwise.} \end{cases}$ --- $\int 24y^2/y^2 \, dy$ with limits $=12$	M1 A1 A1 A1 B1 M1 A1 **7** --- M1 A1 **2** **[9]**	 Use of F $8y^3$ obtained correctly Correct range. Condone omission of $y \le 0$ For G'(y) Correct answer with range √ --- With attempt at integration
5(i) **(ii)**	Use $p_s \pm z\sqrt{(p_s q_s/200)}$ z=1.645 $s = \sqrt{(0.135 \times 0.865/200)}$ (0.0952, 0.1747) --- $H_0: p_1 - p_2 = 0$, $H_1: p_1 - p_2 > 0$ $\frac{27/200 - 8/100}{\sqrt{35/300 \times 265/300 \times (200^{-1} + 100^{-1})}}$ = 1.399 > 1.282 Reject H_0. There is sufficient evidence at the 10% significance level that the proportion of faulty bars has reduced	M1 B1 A1 A1 **4** --- B1 M1 B1 A1 A1 M1 A1 **7** **[11]**	 Or /199 (0.095, 0.175) to 3DP --- Or equivalent Correct form. Pooled estimate of p = 35/300 Correct form of sd OR: P($z \ge 1.399$) = 0.0809 <0.10 **SR: No pooled estimate: B1M1B0B0** **A1 for 1.514, M1A1 Max 5/7**
6(i) **(ii)**	Assumes that decreases have a normal distn $H_0: \mu_{O-F} = 0.2$ (or $\ge$), $H_1: \mu_{O-F} > 0.2$ O-F: 0.6 0.4 0.2 0.1 0.3 0.2 0.4 0.3 $\bar{D} = 0.3125$ $s^2 = 0.024107$ (0.3125-0.2)/√(0.024107/8) =2.049 > 1.895 Reject H_o – there is sufficient evidence at the 5% significance level that the reduction is more than 0.2 --- $0.3125 \pm t \sqrt{(0.024107/8)}$ t = 2.365 (0.1827, 0.4423)	B1 B1 M1 B1 A1 M1 A1 M1 A1 **9** --- M1 B1 A1 **3** **[12]**	B1 Use paired differences t-test Must have /8 OR: P($t \ge 2.049$)= 0.0398 <0.05 Allow M1 from t_{14} = 1.761 **SR: 2-sample test:B1B1M0B1A0** **M1 using 1.761 A0 Max 4/9** --- Allow with z but with /8 Rounding to (0.283, 0.442)

7(i)	H_0:Vegetable preference is independent of gender H_1: All alternatives	 B1	 For both hypotheses
	E-Values 26 16.25 22.75 22 13.75 19.25	M1 A1	At least one correct All correct
	$\chi^2 = 5^2(26^{-1} + 22^{-1}) + 7.25^2(16.25^{-1} + 13.75^{-1})$ $+ 2.25^2(22.75^{-1} + 19.25^{-1})$	M1 A1	Correct form of any one All correct
	$=9.641$	A1	ART 9.64
	$9.64 > 5.991$	M1	OR: $P(\geq 9.641)=0.00806 < 0.05$
	Reject H_0, (there is sufficient evidence at the 5% that) vegetable preference and gender are not independent	A1 **8**	
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(ii)	- (H_0: Vegetables have equal preference H_1: All alternatives)	 M1	
	Combining rows: 48 30 42 E-Values: 40 40 40	A1 M1	
	$\chi^2 = (8^2+10^2 + 2^2)/40$ $= 4.2$	A1	
	$4.2 < 4.605$	M1	OR:$P(\geq 4.2) = 0.122 > 0.10$
	Do not reject H_0, there is insufficient evidence at the 10% significance level of a difference in the proportion of preferred vegetables	A1 **6** **[14]**	AEF in context